MATHEMATICS METHODS

MELVILLE
SENIOR HIGH SCHOOL

YEAR 12, UNIT 4

Semester 2, 2021
Test 4
Weighting 7%

[Curriculum references: Logarithms 4.1.1- 4.1.14]

Section One: Calculator–free

Student Name: Solutions

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: one minute

Working time for paper: forty minutes

Total Marks: forty-six marks

Materials required/recommended for this section
To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be allocated for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. It is recommended that students **do not use a pencil,** except in diagrams

Question 1. [2, 2 = 4 marks]

Express each of the following as a single logarithm.

a) $2\log 3 + \log 2 - \log 6$

$= \log 9 + \log 2 - \log 6$

$= \log\left(\frac{9 \times 2}{6}\right)$

$= \log 3$ ✓✓

b) $(\log x)^3 \div (\log x)^2 + \log x^2$

$= \log x + 2\log x$

$= 3\log x$ ✓✓

Question 2. [2, 2 = 4 marks]

Evaluate each of the following, showing full working.

a) $1.5\log_8 4$

$= \log_8 4^{3/2}$

$= \log_8 (2^2)^{3/2}$

$= \log_8 8$

$= 1$ ✓✓

b) $\frac{\log 135 - \log 5}{\log 3^2}$

$= \frac{\log\left(\frac{135}{5}\right)}{2\log 3}$

$= \frac{\log 27}{2\log 3}$

$= \frac{3\log 3}{2\log 3} = \frac{3}{2}$ ✓✓

Question 3. [1, 3 = 4 marks]

If $\log x = y$, where x is positive, express each of the following in terms of y.

a) $\log x^2$

$= 2y$ ✓

b) $\log xm^3 - 3\log m$

$= \log x + 3\log m - 3\log m$ ✓

$= \log x$

$= y$ ✓✓

Question 4. [3, 3 = 6 marks]

Give an exact value for x in terms of $\log_{10}$ (simplify your logs)

$4^{2x} - 4^x - 6 = 0$

Let $y = 4^x$

$y^2 - y - 6 = 0$ ✓

$(y-3)(y+2) = 0$

$y = 3$ or $y = -2$ ✓

$4^x = 3$ or $4^x = -2$

↑ reject

$x = \frac{\log 3}{\log 4}$ ✓

b) $2^{x-3} = 5^{2x+1}$

$2^x \cdot 2^{-3} = 5^{2x} \cdot 5^1$

$\frac{2^x}{5^{2x}} = \frac{5}{2^{-3}}$

$\left(\frac{2}{25}\right)^x = 40$ ✓

$\log_{\frac{2}{25}} 40 = x$ ✓

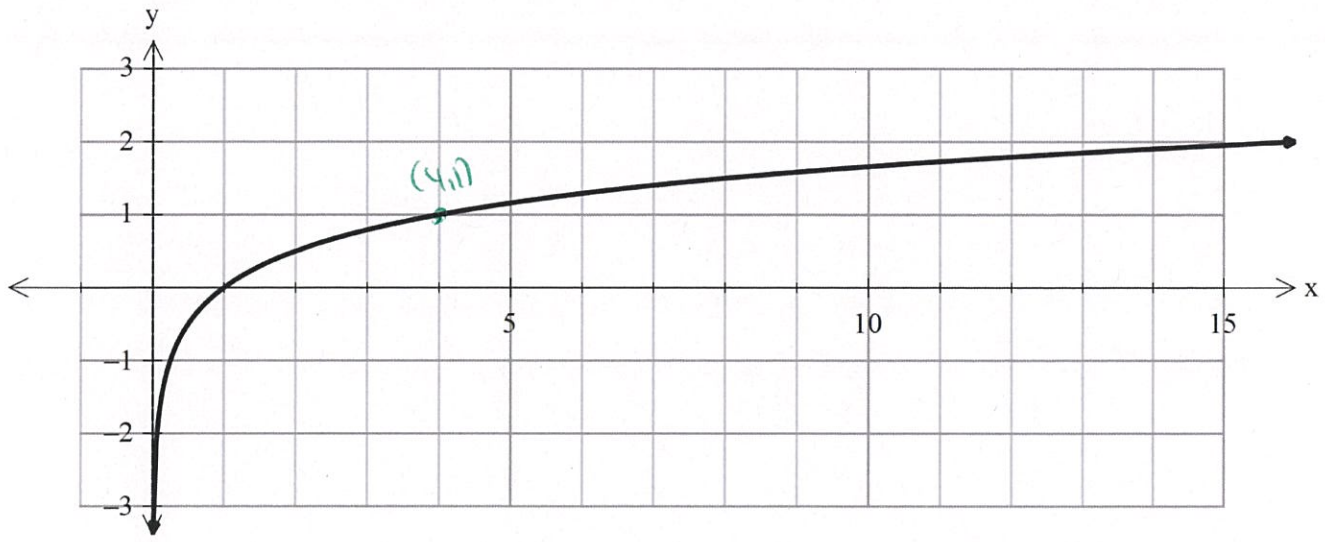
$\therefore x = \frac{\log 40}{\log \frac{2}{25}}$

$= \frac{\log 40}{\log 2 - 2\log 5}$ ✓

or $\frac{\log 5 + 3\log 2}{\log 2 - 2\log 5}$

Question. 5. [1, 1, 1, 2 = 5 marks]

The function $y = \log_a x$ is graphed below. Use the graph to answer the questions that follow.



a) Find the value of a.

$\log_a 4 = 1$

$a = 4$

b) Use the graph to approximate to 1 d.p. the solution to $\log_a x = 1.6$

$x \approx 9$ (within ± 0.5)

c) Approximate the solution for x if $x = a^{1.2}$

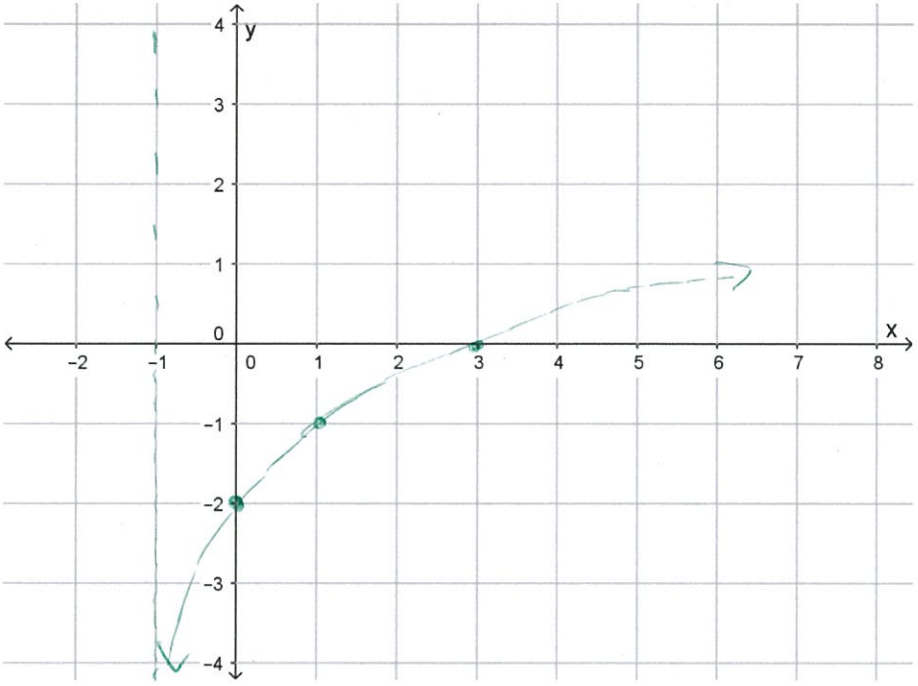
$x \approx 5$

d) Use the graph to find an approximation to $\log_3 7$

$= \frac{\log 7}{\log 3}$ ✓ $\approx \frac{1.4}{0.8} = \frac{14}{8}$ ✓

Question. 6 [3 marks]

Sketch the graph of $y = \log_2(x + 1) - 2$. Clearly label the key features.



$x = -1$

x int $(3, 0)$
y int $(0, -2)$ } ✓

$x = -1$ asymptote ✓

general shape ✓

Question. 7. [3, 3, 3 = 9 marks]

Differentiate each of the following with respect to x, showing full working.

(DO NOT SIMPLIFY YOUR ANSWER)

a) If $y = x^2 ln(sin\ x)$ find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 2x\ln\sin(x) + \frac{x^2\cos x}{\sin x}$$

b) $g(x) = \ln[(x^2+1)(x-1)^4]$

$$g(x) = \ln(x^2+1) + \ln(x-1)^4$$

$$= \ln(x^2+1) + 4\ln(x-1)$$

$$g'(x) = \frac{2x}{x^2+1} + \frac{4}{(x-1)}$$

c) $y = \log_2(x^3 - 2x)$

$$y = \frac{\ln(x^3-2x)}{\ln 2}$$

$$\frac{dy}{dx} = \frac{1}{\ln 2} \cdot \frac{(3x^2-2)}{(x^3-2x)}$$

Question 8. [2, 3 = 5 marks]

Integrate each of the following, showing full working.

a) $\int \frac{x}{x^2-4}\,dx$ $= \frac{1}{2}\int \frac{2x}{x^2-4}\,dx$ ✓

$= \frac{1}{2}\ln|x^2-4| + C$ ✓

b) $\int \frac{4x^4-3x^2+x}{x^3}\,dx$ $= \int 4x - \frac{3}{x} + \frac{1}{x^2}\,dx$ ✓

$= 2x^2 + 3\ln|x| - \frac{1}{x} + C$

✓ ✓

−1 no +C

Question 9. [2, 4 = 6 marks]

Given $y = \frac{\ln x}{x}$

a) Find $\frac{dy}{dx}$ $= \dfrac{x \cdot \frac{1}{x} - \ln x \cdot (1)}{x^2}$ ✓

$= \dfrac{1 - \ln x}{x^2}$ ✓

b) Hence, or otherwise, use $\frac{dy}{dx}$ to show that $\int_1^2 \frac{\ln x}{x^2} dx = \frac{1 - \ln 2}{2}$

If $\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$

$\frac{dy}{dx} = \frac{1}{x^2} - \frac{\ln x}{x^2}$.

$\frac{\ln x}{x^2} = \frac{1}{x^2} - \frac{dy}{dx}$

$\int \frac{\ln x}{x^2} dx = \int_1^2 \frac{1}{x^2} - \frac{dy}{dx} dx$ ✓

If $x = 1$, $y = 0$

$x = 2$, $y = \frac{1}{2} \ln 2$

$= \int_1^2 \frac{1}{x^2} dx - \int_0^{\frac{\ln 2}{2}} dy$ ✓

$= \left[-\frac{1}{x}\right]_1^2 - \left[y\right]_0^{\frac{\ln 2}{2}}$ ✓

$= -\frac{1}{2} + 1 - \frac{\ln 2}{2}$

$= \dfrac{1 - \ln 2}{2}$ ✓

End of Calculator Free Section ☺

Additional working space.

Additional working space.

Qu 9b Alternative Soln

$$\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1}{x^2} - \frac{\ln x}{x^2}$$

$$\int_1^2 \frac{d}{dx}\left(\frac{\ln x}{x}\right) dx = \int_1^2 \frac{1}{x^2}\, dx - \int_1^2 \frac{\ln x}{x^2}\, dx \checkmark$$

$$\frac{\ln 2}{2} - \frac{\ln 1}{1} = \left[\frac{x^{-1}}{-1}\right]_1^2 - \int_1^2 \frac{\ln x}{x^2}\, dx$$

$$\int_1^2 \frac{\ln x}{x^2}\, dx = \left[-\frac{1}{x}\right]_1^2 - \left(\frac{\ln 2}{2} - 0\right) \checkmark$$

$$\int_1^2 \frac{\ln x}{x^2}\, dx = -\frac{1}{2} - \left(\frac{-1}{1}\right) - \frac{\ln 2}{2} \checkmark$$

$$= -\frac{1}{2} + 1 - \frac{\ln 2}{2}$$

$$= \frac{1}{2} - \frac{\ln 2}{2}$$

$$= \frac{1 - \ln 2}{2} \checkmark$$